請將答案寫在 答題紙 上。

All answers should be written on the ANSWER SHEET.

甲部：每題 4 分
Section A – each question carries 4 marks

1) 若一正整數和它的正因數之和的和為 4035。求該數的最大值。
   If the sum of a positive integer and the sum of its positive divisors is 4035, find the largest value of the number.

2) 已知計算 $m \times a$ 時，若分別將 a 取至最接近十分位和百分位，兩積之差的最大值為 50。求 m 的值。
   It is given that, when finding the value of $m \times a$, if a is rounded to the nearest tenth and hundredth, the maximum difference between two products is 50. Find the value of m.

3) 若 x、y、z 符合下列方程組，求 $x + y + z$ 的值。
   If x, y and z satisfy the following system of equations, find the value of $x + y + z$.

4) 有多少種方法從所有 4 位奇數中選出 4 個數字使得它們的和為 10001？
   How many ways are there to choose 4 numbers from all 4-digit odd numbers such that their sum is 10001?

5) 關於 x 和 y 的方程 $20x + 17y = 2017$，其中一對正整數解顯然是 $(x, y) = (100, 1)$。求該方程的正整數解組數目。
   For the equation $20x + 17y = 2017$ about x and y, one trivial pair of positive integral solution is $(x, y) = (100, 1)$. Find the number of pairs of positive integral solutions to the equation.

6) 圖 6 中，$ABCDEFGH$ 為正八邊形，且 ΔFJE 為等邊三角形，若反角 $\angle GJD = x°$，求 x 的值。
   In Figure 6, $ABCDEFGH$ is a regular octagon and ΔFJE is an equilateral triangle. If reflex $\angle GJD = x°$, find the value of x.
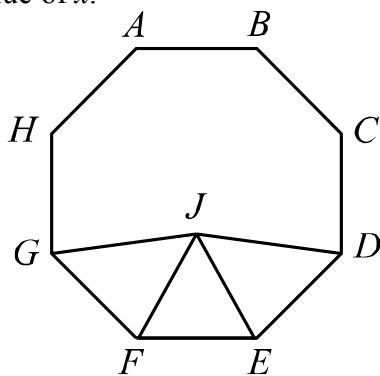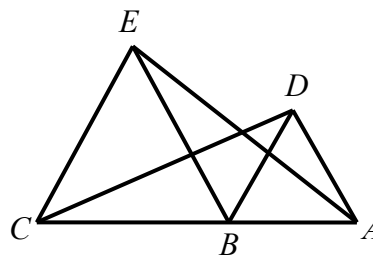
圖 6
Figure 6

圖 7
Figure 7

7) 圖 7 中，ABC 是直線。ΔBCE 和 ΔABD 是等邊三角形。若 $\angle DCB = 22°$ 且 $\angle AEB = x°$，求 x 的值。
   In Figure 7, ABC is a straight line. ΔBCE and ΔABD are equilateral triangles. If $\angle DCB = 22°$ and $\angle AEB = x°$, find the value of x.

8) 圖 8 中，*AB* // *CD*、*BD* = *AC*。若 Δ*ABE* 和 Δ*CDE* 的面積分別是 8 和 18。求 Δ*ACE* 的面積。

In Figure 8, *AB* // *CD* and *BD* = *AC*. If the areas of Δ*ABE* and Δ*CDE* are 8 and 18 respectively, find the area of Δ*ACE*.
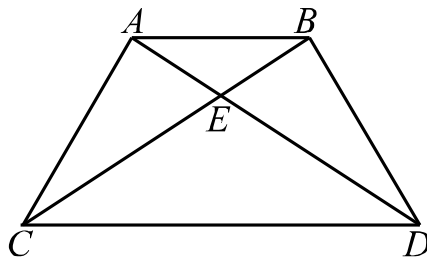
圖 8
Figure 8

~ 甲部完 ~
~ End of Section A ~

請將答案寫在 答題紙 上。

All answers should be written on the ANSWER SHEET.

乙部：每題 5 分

Section B – each question carries 5 marks

9) 求以下算式的值。
Find the value of the following expression.
$3^2 + 8^2 + 13^2 + 18^2 + ... + 98^2$

10) 已知三角形三角成一等差數列，且第二大的角的對邊為該三角形的最短邊，求該三角形的最長邊與最短邊的最大比。
It is given that the three angles of a triangle form an arithmetic sequence and the opposite side of the second largest angle is the shortest side of the triangle. Find the greatest ratio of the longest side of the triangle to the shortest side.

11) 若 x、y 均為整數且 $xy - x + y = 6$，求 $x + y$ 的最大值。
If x and y are integers and $xy - x + y = 6$, the largest value of $x + y$.

12) 若 $2018x$ 除以 2016 的餘數為 44，求 x 的最小正整數值。
If the remainder of $2018x$ divided by 2016 is 44, find the smallest positive integral value of x.

13) 以下算式謎中，A、C、F 均為一位正整數；B、D、E 為一位非負整數。求此算式謎解的數目。
In the cryptarithm below, A, C and F are all one-digit positive integer; B, D and E are one-digit non-negative integers. Find the number of solutions to the cryptarithm.

$$
\begin{array}{r}
A\ B \\
+\ \ C\ D \\
\hline
F\ 0\ E
\end{array}
$$

14) m 個連續正整數的平均數為 a。若將該些數中最小的數換成 960，或將該數的最大數換成 990，他們的平均數會變成 75。求 a 的值。
The average of m consecutive positive integers is a. If the smallest number among these numbers is substituted by 960, or the largest number among these numbers is substituted by 990, their average becomes 75. Find the value of a.

15) 在圖 15 中，有多少種方法在這個 6×3 的方格上選取 3 格，使得該 3 格沒有任何兩格位於同一行或同一列？
In Figure 15, how many ways are there to pick 3 grids from the 6×3 grids, such that these 3 grids do not have any two placing in the same row or column?
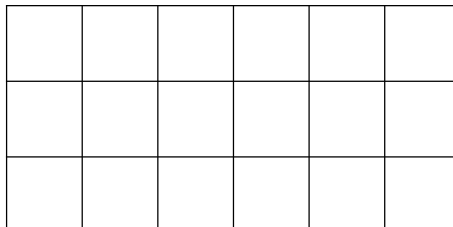

圖 15

Figure 15

16) 若 a、b、c 為整數且符合 $(x+a)(x+b)(x+c) \equiv x^3 + kx + 6$。求 k 的值。

    If a, b and c are integers and satisfy $(x+a)(x+b)(x+c) \equiv x^3 + kx + 6$, find the value of k.

~ 乙部完 ~
~ End of Section B ~

丙部：每題 7 分
Section C – each question carries 7 marks

17) 圖 17 中，長方形 *ABCD* 的長、闊分別為 72 和 30。*G* 為 *BC* 上的一動點。若 *AG* + *GE* 的最小可能值為 90，*AE* 和 *DB* 相交於 *F*。求 *DF* 的值。

In Figure 17, the length and width of rectangle *ABCD* are 72 and 30 respectively. *G* is a moving point on *BC*. If the smallest possible value of *AG* + *GE* is 90, *AE* and *DB* intersects at *F*, find the length of *DF*.
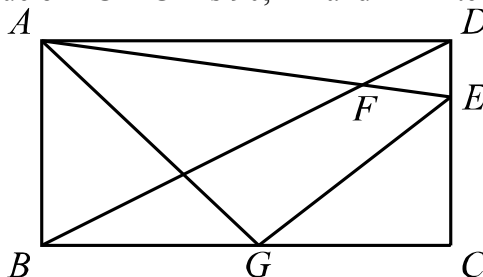

圖 17
Figure 17

18) 在圖 18 中，$BD:DE:EC = 2:2:3$、*F* 為 *AC* 的中點、*G* 為 *BF* 與 *AD* 的交點、*H* 為 *BF* 與 *AE* 的交點。若 ΔABC 的面積為 693，求四邊形 *DEHG* 的面積。

In Figure 18, $BD:DE:EC = 2:2:3$, *F* is the mid-point of *AC*, *G* is the intersecting point of *BF* and *AD*, and *H* is the intersecting point of *BF* and *AE*. If the area of ΔABC is 693, find the area of quadrilateral *DEHG*.
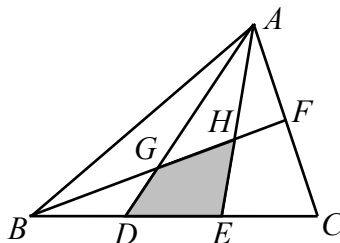

圖 18
Figure 18

19) 若 $x^2(y-z) + y^2(z-x) + z^2(x-y)$ 能被因式分解為三個一次多項式之積，求該三個一次多項式的所有係數之和。

If $x^2(y-z) + y^2(z-x) + z^2(x-y)$ can be factorized as a product of three linear polynomials, find the sum of all coefficients of these three linear polynomials.

20) *k* 和 *d* 是正整數，且 *k* 和 *k* + 2*d* 均為兩位質數。若 $k+(k+1)+\ldots+(k+2d)$ 為 18 的倍數，求 *k* + *d* 的最小值。

k and *d* are positive integers while *k* and $k+2d$ are both two-digit prime numbers. If $k+(k+1)+\ldots+(k+2d)$ is a multiple of 18, find the smallest value of $k+d$.

~ 全卷完 ~
~ End of Paper ~