

請將答案寫在 答題紙 上。

All answers should be written on the ANSWER SHEET.

甲部：每題 4 分

Section A – each question carries 4 marks

- 1) 圖 1 中， E 和 F 分別為 AC 和 AB 上的點， X 是 BE 和 CF 的交點。若 $AF:FB = 2:1$ 、 $AE:EC = 1:1$ ，求 $FX:XC$ 。

In Figure 1, E and F are points on AC and AB respectively. X is the intersecting point of BE and CF . If $AF:FB = 2:1$, $AE:EC = 1:1$, find $FX:XC$.

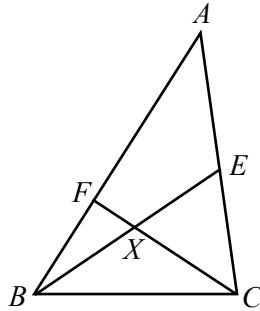


圖 1
Figure 1

- 2) 若 $\frac{\sin \alpha}{\sin \beta} = 2017$ 及 $\frac{\cos \beta}{\cos \alpha} = 311$ ，求 $\frac{\sin 2\alpha}{\sin 2\beta}$ 的值。
If $\frac{\sin \alpha}{\sin \beta} = 2017$ and $\frac{\cos \beta}{\cos \alpha} = 311$, find the value of $\frac{\sin 2\alpha}{\sin 2\beta}$.

- 3) 若 x 是正數而且 $x = 3 + \frac{1}{6 + \frac{1}{6 + \frac{1}{6 + \dots}}}$ ，求 x 的值。

If x is positive and $x = 3 + \frac{1}{6 + \frac{1}{6 + \frac{1}{6 + \dots}}}$, find the value of x .

- 4) 求以下方程的正整數解的數目。
Find the number of positive integral solutions of the following equation.
 $a + b + c + d = 17$

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- 5) 圖 5 中，兩直線為兩圓的公切線且夾角為 60° ，大圓與小圓的半徑之比為 $k:1$ ，求 k 的值。
In Figure 5, the two straight lines are the common tangents to the two circles with included angle of 60° . The ratio of the radii of the larger and smaller circles is $k:1$. Find the value of k .

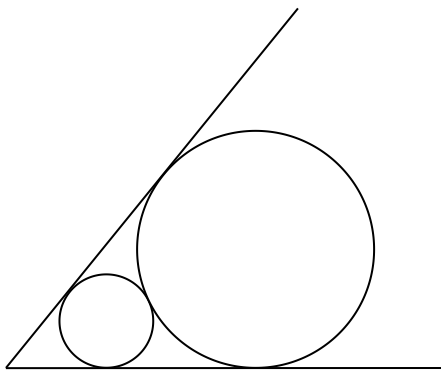


圖 5

Figure 5

- 6) 求邊長為 1 的正八面體的體積。
Find the volume of a regular octahedron (i.e. 8-faced polyhedron) with side length of 1.
- 7) 已知一點 (x_0, y_0) 與一直線 $ax + by + c = 0$ 之間的距離等於 $\frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$ ，求拋物線 $y = (x - 20)^2 + 17$ 與直線 $4x + y + 311 = 0$ 之間的最短距離。
Given that the distance between a point (x_0, y_0) and a straight line $ax + by + c = 0$ is $\frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$. Find the shortest distance between parabola $y = (x - 20)^2 + 17$ and straight line $4x + y + 311 = 0$.
- 8) 求方程 $x^2 - 5x - 4\sqrt{x^2 - 5x - 5} - 2 = 0$ 的根的平方和。
Find the sum of squares of the roots of equation $x^2 - 5x - 4\sqrt{x^2 - 5x - 5} - 2 = 0$.

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~ End of Section A ~

請以最簡形式填寫答案。若計算結果是分數，請化至最簡，並確保為真分數或帶分數，或將計算結果寫成小數。

答案可以根式表示，唯該根式必須是最簡形式。除特別註明外，毋需填寫單位。錯誤單位將不給予任何分數。

Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. You may use square root to represent the answer which is in the simplest form.

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乙部：每題 5 分

Section B – each question carries 5 marks

- 9) 若 $1+\sqrt{3}i$ 是實係數二次方程 $x^2+ax+b=0$ 的一個根，求 $a+b$ 的值。
If $1+\sqrt{3}i$ is a root of the quadratic equation $x^2+ax+b=0$ with real coefficients. Find the value of $a+b$.
- 10) 若 $[x]$ 為不大於 x 的最大整數， $\{x\}$ 為 $x-[x]$ 的值，求方程 $2x+3\{x\}=4[x]$ 的所有解之和。
If $[x]$ is the greatest integer not greater than x and $\{x\}$ is the value of $x-[x]$, find the sum of all solutions of the equation $2x+3\{x\}=4[x]$.
- 11) 若 x 和 y 為整數且 $2x-3y=xy+2017$ ，求解的組數。
If x and y are integers and $2x-3y=xy+2017$, find the number of sets of solutions.
- 12) 設 x 為整數且 $0 \leq x < 11$ ，若 11 整除 $6x^2-x+9$ ，求 x 的可能值之和。
Let x be an integer and $0 \leq x < 11$. If $6x^2-x+9$ is divisible by 11, find the sum of all possible values of x .
- 13) 已知 $10! = 3628800$ ，求以下算式的值。
Given that $10! = 3628800$. Find the value of the following expression.
$$\sum_{i=1}^8 \frac{1}{(i+2)i!}$$
- 14) 若 α 和 β 是方程 $[\log(6x)] \left[\log\left(\frac{x}{1866}\right) \right] = 2017$ 的兩個相異實根，求 $\alpha\beta$ 的值。
If α and β are two distinct real roots of equation $[\log(6x)] \left[\log\left(\frac{x}{1866}\right) \right] = 2017$, find the value of $\alpha\beta$.
- 15) 已知 $(\cos \theta + i \sin \theta)^n = \cos(n\theta) + i \sin(n\theta)$ ，以 $a+bi$ 的形式表示 $\frac{(1+i)^{2017}}{8^{311} \times 2^{16} \times 4^{30}}$ ，當中 a 和 b 是實數。
Given that $(\cos \theta + i \sin \theta)^n = \cos(n\theta) + i \sin(n\theta)$. Express $\frac{(1+i)^{2017}}{8^{311} \times 2^{16} \times 4^{30}}$ in the form of $a+bi$, where a and b are real numbers.

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16) 在 1 至 2017 這 2017 個正整數中隨機選取 n 個，則其中必定有兩個數 a 和 b 使得 $\frac{a}{b} \leq \frac{b}{a} \leq 2$ ，求 n 的最小值。

If n numbers are picked at random among the 2017 positive integers from 1 to 2017, then there must be two numbers a and b such that $\frac{a}{b} \leq \frac{b}{a} \leq 2$. Find the least possible value of n .

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~ End of Section B ~

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丙部：每題 7 分

Section C – each question carries 7 marks

- 17) 若 a 、 b 和 c 為非負數，且 $a+b+c=10$ ，求 $abc+3ab-2ac+bc-6a+3b-2c$ 的最大值。
If a , b and c are non-negative numbers and $a+b+c=10$, find the largest possible value of $abc+3ab-2ac+bc-6a+3b-2c$.
- 18) 老師分配 6 份不同的任務給 4 名同學，當中每份工作都只分配給一名同學，而每名同學都有收到任務，問有多少種分配方法？
A teacher assigns 6 different tasks to 4 students, where each task is to be assigned to one student only and all students are assigned tasks. How many ways of assignment are there?
- 19) 已知一位孩子是聰明的話，他是頑皮的概率是 0.7；已知一位孩子是不聰明的話，他是頑皮的概率是 0.3。隨機一位孩子是頑皮的概率是三分之一。已知一位孩子是頑皮的，求他是聰明的概率。
Given that a child is intelligent, the probability of him being naughty is 0.7. Given that a child is not intelligent, the probability of him being naughty is 0.3. The probability of a child chosen at random being naughty is $\frac{1}{3}$. Given that a child is naughty, find the probability of him being intelligent.
- 20) 圖 20 中 $BCDE$ 是圓內接四邊形， ABC 和 AED 是直線。已知 $AE=28$ 、 $AB=35$ 、 $DC=60$ 、 $AD \perp EB$ ，求 EC 的長度。
In Figure 20, $BCDE$ is a cyclic quadrilateral. ABC and AED are straight lines. Given that $AE=28$, $AB=35$, $DC=60$ and $AD \perp EB$, find the length of EC .

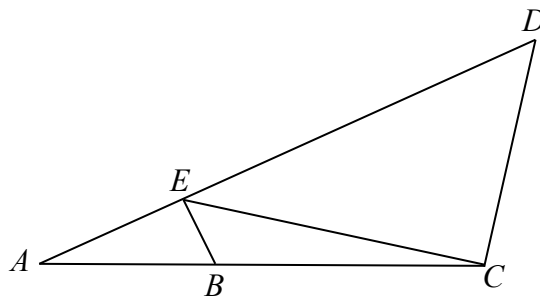


圖 20

Figure 20

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