Question 1 (Distributive property)

Let M denotes an integer that is equal to

$$\frac{(1+2+3+\ldots+10)^2-1-2-3-\ldots-10}{1+2+3+\ldots+10}.$$

What is the value of M?

A. 53
B. 54
C. 55
D. 56
E. None of the above

Ans: B

Question 2 (Simple algebra)

If $\sqrt{n^3+n^3+n^3+n^3+n^3+n^3}=36$, then what is the value of n?

A. 3
B. 4
C. 5
D. 6
E. None of the above

Ans: D

Question 3 (Mathematical definitions)

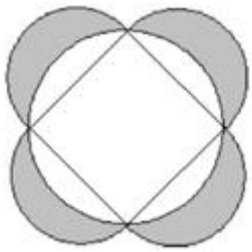
Which of the following is false?

A. 119 is not a prime number.
B. For all integers x, y, z, if $xz=yz$, then $x=y$.
C. The equation $2+x=2$ has no positive integer solution.
D. For all integers m, n, p, if $m+p=n+p$, then $m=n$.
E. The number 0 is an integer.

Ans: C

Question 4 (Geometry, Circles)

A square is inscribed in a circle with diameter 4. Four smaller circles are then constructed with their diameters on each of the sides of the square. Determine the shaded area.



A. 1
B. 2
C. 4
D. 8
E. 16

Ans: D

Question 5 (Statistics)

The following values are NOT arranged in decreasing nor increasing order:

$24, x, 46, 2x, 17, x + 7, 35$. If the median of these values is 25, what is the value of x?

A. 19
B. 16
C. 20
D. 17
E. 18

Ans: E

Question 6 (Parabola)

The path of a particle can be modelled by $H = -16t^2 + 8t + 4$, where H is the height in micrometre of the particle t microseconds after it is released. What is the maximum height, in micrometre, reached by the particle?

A. 3
B. 4
C. 5
D. 6
E. None of the above

Ans: C

Question 7 (Work Rate)

A product is made up of 3 components: X, Y and Z. A worker each day can produce 5 component X or 3 component Y or 6 component Z. Given that the factory has 210 workers, how many workers should be working on component X to maximize the production of the product?

A. 50
B. 60
C. 70
D. 75
E. 80

Ans: B

Question 8 (Calculation, algebra)

"Fermat's Theorem on Sums of Two Squares" states that an odd prime number, such as 2017, can be expressed as a sum of squares of 2 integers:

$$2017 = x^2 + y^2$$

if the prime number is 1 more than a multiple of 4.

In the above equation, find the value of x if the value of y is 10 less than 6 times of x.

A. 3
B. 9
C. 41
D. 44
E. None of the above

Ans: B

Question 9 (Counting)

In how many ways can you walk up a flight of stairs which has 7 steps if you can take 1 or 2 steps at a time?
(For example, you can walk up a flight of stairs which has 3 steps in 3 different ways: 1-1-1, 1-2 or 2-1)

A. 22
B. 21
C. 20
D. 19
E. None of the above

Ans: B

Question 10 (Geometry concepts)

How many of the following statements is/are true?

- In a triangle whose 2 interior angles are x and y, it is possible to have $\cos x = \cos y = 0$.
- If a triangle is inscribed in a semicircle, then the triangle must be a right-angled triangle.
- There exists a triangle whose sides are 10 cm, 20 cm and 30 cm.
- If the perimeter of a rectangle is given, then the largest area can be obtained if all sides are equal.

- If the diagonals of a parallelogram bisect each other, then this quadrilateral must be a rhombus.

A. 1
B. 2
C. 3
D. 4
E. 5

Ans: A

Question 11 (Power)

Which one of the following has the least value?

A. 2^{40}
B. 3^{30}
C. 5^{20}
D. 10^{15}
E. 31^{8}

Ans: E

Question 12 (Modulus, factors)

How many ordered pairs of integers (x, y) are there such that $|xy| = 144$?

A. 32
B. 40
C. 48
D. 56
E. 60

Ans: E

Question 13 (Counting)

A number is written with one 1, followed by three 3s, then five 5s, then seven 7s,
then nine 9s, then eleven 11s, etc. Thus it begins with 13335555577... . What is its 75th digit?

A. 1
B. 3
C. 5
D. 7
E. 9

Ans: C

Question 14 (Factors)

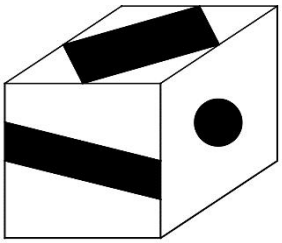
What is the smallest positive integer n such that $n!$ is a multiple of 10^7?

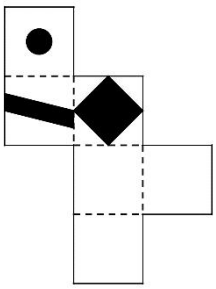
A. 15
B. 20
C. 25
D. 30
E. None of the above

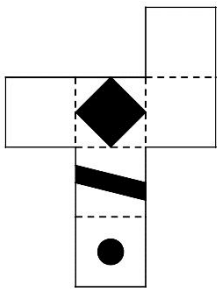
Ans: D

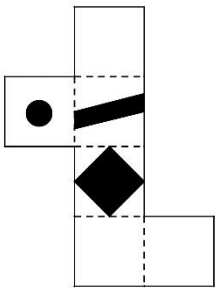
Question 15 (Spatial)

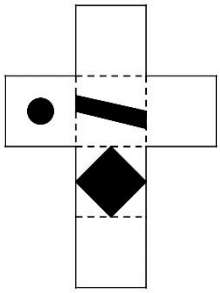
Which one of the following options can be folded to form the cube shown in the diagram below?



A. 

B. 

C. 

D. 

E. None of the above

Ans: D

Question 16 (Factors, Counting)

The numbers from 1 to 2017 are listed in the following order: First all numbers which are *not* divisible by 3 are listed in increasing order. Then all numbers which are divisible by 3 but *not* by 3^2 are listed in increasing order. Then all numbers which are divisible by 3^2 but *not* by 3^3 are listed in increasing order, and so on. What is the last number on this list? (Give the entire number, not just its last digit.)

Ans: 1458

Question 17 (Tangent-radius theorem)

The hypotenuse of a right-angled triangle is 20 cm and the radius of the inscribed circle is 4 cm. The perimeter of the triangle in cm is _____

Ans: 48

Question 18 (Number theory)

How many two-digit positive integers N have the property that the sum of N and the number obtained by reversing the order of the digits of N is a perfect square?

Ans: 8

Question 19 (Pattern, Logic, Number theory)

What is the largest number of zeros that can occur at the end of $1^n + 2^n + 3^n + 4^n$ for any positive integer n?

Ans: 2

Question 20 (Logic)

Geoff is trying to unlock the forgotten passcode for his luggage. The passcode consists of 3 digits, each of which is from 0 to 9.

He tried 923, only one number is correct and correctly placed.
He tried 958, only one number is correct but wrongly placed.
He tried 379, only two numbers are correct but wrongly placed.
He tried 142, none of the numbers is correct.
He tried 127, only one number is correct but wrongly placed.

What is the passcode?

Ans: 783

Question 21

Evaluate the expression below.

$$\frac{100}{1+2}+\frac{100}{1+2+3}+\frac{100}{1+2+3+4}+\frac{100}{1+2+3+4+5}+\ldots+\frac{100}{1+2+3+\ldots+99}$$

Ans: 98

Question 22 (Probability)

There are 100 balls in a box. A ball in the box is either black or white. There are 67 black balls and 33 white balls. A ball in this box is either made up of plastic or rubber. Of the 100 balls, 34 are plastic balls and 66 are rubber balls. Moreover, 19 plastic balls are white. The probability of choosing a black or rubber ball is $\frac{m}{100}$. What is m?

Ans: 81

Question 23 (Pattern, Telescopic Sum)

Evaluate the expression below.

$$9700\times\left(\frac{1}{1\times4\times7}+\frac{1}{4\times7\times10}+\frac{1}{7\times10\times13}+\ldots+\frac{1}{94\times97\times100}\right)$$

Ans: 404

Question 24 (Algebraic manipulation)

Let a,b and c be non-zero real numbers such that $a+\frac{1}{b}=6$, $b+\frac{1}{c}=10$ and $c+\frac{1}{a}=14$.

Find $abc+\frac{1}{abc}$.

Ans: 810

Question 25 (Logarithm)

Simplify

$$\frac{1}{\log_2 K}+\frac{1}{\log_3 K}+\frac{1}{\log_4 K}+\ldots+\frac{1}{\log_{500} K},$$

where $K=(500!)^{\frac{1}{5000}}$

Ans: 5000