Question 1 (Mathematical definitions)

Which one of the following is false?

A. 91 is not a prime number.
B. The equation $2 + x = 2$ has no positive integer solution.
C. For all integers x, y, z, if $xz = yz$, then $x = y$.
D. For all integers m, n, p, if $m + p = n + p$, then $m = n$.
E. The number -1 is an integer.

Ans: C

Question 2 (Simple algebra)

If $\sqrt{n^3 + n^3 + n^3 + n^3 + n^3} = 25$, then what is the value of n?

A. 2
B. 3
C. 4
D. 5
E. None of the above

Ans: D

Question 3 (Counting, Pattern)

A number is written with one 1, followed by three 3s, then five 5s, then seven 7s, then nine 9s, then eleven 11s, etc. Thus, it begins with 13335555577…. What is its 49th digit from the left of the number?

A. 1
B. 3
C. 5
D. 7
E. 9

Ans: B

Question 4 (Speed)

An SMRT bus travels at 50km/h. A car starts 12 km behind the bus and travels at 80km/h. How many minutes later will the car catch up with the SMRT bus?

A. 24 minutes
B. 12 minutes
C. $\frac{72}{13}$ minutes
D. 150 minutes
E. 6 minutes

Ans: A

Question 5 (Statistics)

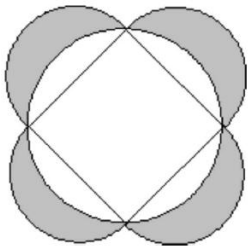
In a list of 15 numbers, which one of the following CANNOT affect the value of the median?

A. Multiplying all numbers by 0.5
B. Decreasing each number by 1
C. Introducing a 16^{th} number
D. Increasing the largest number
E. Eliminating the first number

Ans: D

Question 6 (Geometry, Circles)

A square is inscribed in a circle with diameter 2. Four smaller circles are then constructed with their diameters on each of the sides of the square. Determine the shaded area.



A. 1
B. 2
C. 4
D. 8
E. 16

Ans: B

Question 7 (Modulus, factors, counting)

How many ordered pairs of integers (x, y) are there such that $|xy| = 36$?

A. 9
B. 20
C. 32
D. 34
E. None of the above

Ans: E

Question 8 (Probability)

A box contains exactly 5 chips, three red and two white. Chips are randomly removed one at a time without replacement until all the red chips are drawn or all the white chips are drawn. What is the probability that the last chip drawn is white?

A. $\frac{2}{5}$
B. $\frac{1}{2}$
C. $\frac{3}{5}$
D. $\frac{18}{25}$
E. None of the above

Ans: C

Question 9 (Geometry concepts)

How many of the following statements is/are true?

- In a triangle whose 2 interior angles are x and y, it is possible to have $cos\ x = cos\ y = 0$.
- If a triangle is inscribed in a semicircle, then the triangle must be a right-angled triangle.
- There exists a triangle whose sides are 10 cm, 20 cm and 30 cm.
- If the perimeter of a rectangle is given, then the largest area can be obtained if all sides are equal.
- If the diagonals of a parallelogram bisect each other, then this quadrilateral must be a rhombus.

A. 1
B. 2
C. 3
D. 4
E. 5

Ans: A

Question 10 (Parabola)

Suppose the minimum value $y = (x-3)^2 - \frac{57}{8}$ is P, and the maximum value of $y = -2x^2 + 5x$ is Q, what is the value of $P + Q$?

A. 1
B. 3
C. 4
D. -3
E. -4

Ans: E

Question 11 (Power)

Which one of the following has the greatest value?

A. 2^{40}
B. 3^{30}
C. 5^{20}
D. 10^{15}
E. 31^{8}

Ans: D

Question 12 (Work Rate)

A product is made up of 3 components: X, Y and Z. A worker each day can produce 5 component X or 3 component Y or 6 component Z. Given that the factory has 210 workers, how many workers should be working on component X to maximize the production of the product?

A. 50
B. 60
C. 70
D. 75
E. 80

Ans: B

Question 13 (Counting)

In how many ways can you walk up a flight of stairs which has 7 steps if you can take 1 or 2 steps at a time?

(For example, you can walk up a flight of stairs which has 3 steps in 3 different ways: 1-1-1, 1-2 or 2-1)

A. 19
B. 20
C. 21
D. 22
E. None of the above

Ans: C

Question 14 (Factors)

What is the smallest positive integer n such that $n!$ is a multiple of 100,000?

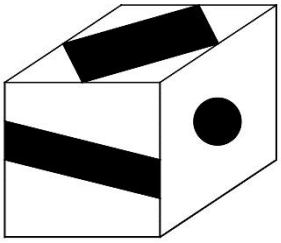
A. 15

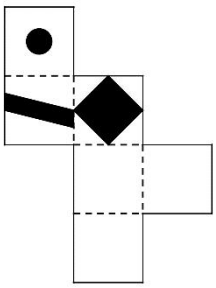
B. 20
C. 25
D. 30
E. None of the above

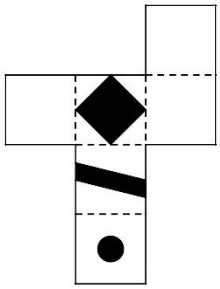
Ans: C

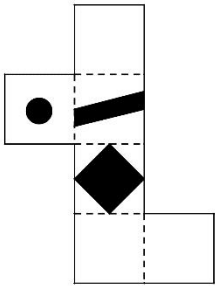
Question 15 (Spatial Visualization)

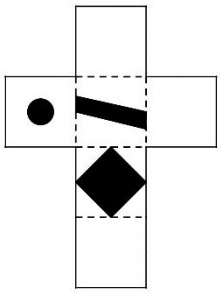
Which one of the following options can be folded to form the cube shown in the diagram below?



A. 

B. 

C. 

D. 

E. None of the above

Ans: D

Question 16 (Pattern)

$P(x)$ is a polynomial function of degree 2. Given that $P(1) = 1$, $P(2) = 5$, and $P(3) = 13$, find the value of $P(4)$.

Ans: 25

Question 17 (Tangent-radius theorem, Pythagoras theorem)

The hypotenuse of a right-angled triangle is 10 cm and the radius of the inscribed circle is 2 cm. What is the perimeter of the triangle (in cm)?

Ans: 24

Question 18 (Number theory)

How many two-digit positive integers N have the property that the sum of N and the number obtained by reversing the order of the digits of N is a perfect square?

Ans: 8

Question 19 (Inequality)

Let c be the maximum integer value of x, and d be the minimum integer value of x in the inequality

$$12 \leq \left|x^2 - 8\right| + 5 \leq 789.$$

What is $c - d$?

Ans: 56

Question 20 (Pattern, Logic, Number theory)

What is the largest number of zeros that can occur at the end of $1^n + 2^n + 3^n + 4^n$ for any positive integer n?

Ans: 2

Question 21 (Pattern, Telescoping series)

Evaluate the expression below.

$$27500 \times \left(\frac{1}{5 \times 10 \times 15} + \frac{1}{10 \times 15 \times 20} + \frac{1}{15 \times 20 \times 25} + \ldots + \frac{1}{45 \times 50 \times 55}\right)$$

Ans: 54

Question 22 (Logic)

Geoff is trying to unlock the forgotten passcode for his luggage. The passcode consists of 3 digits, each of which is from 0 to 9.

He tried 923, only one number is correct and correctly placed.
He tried 958, only one number is correct but wrongly placed.
He tried 379, only two numbers are correct but wrongly placed.
He tried 142, none of the numbers is correct.
He tried 127, only one number is correct but wrongly placed.

What is the passcode?

Ans: 783

Question 23

Evaluate the expression below.

$$\frac{201}{1+2}+\frac{201}{1+2+3}+\frac{201}{1+2+3+4}+\frac{201}{1+2+3+4+5}+\ldots+\frac{201}{1+2+3+\ldots+200}$$

Ans: 199

Question 24 (Algebraic manipulation)

Let a,b and c be non-zero real numbers such that $a+\frac{1}{b}=5$, $b+\frac{1}{c}=12$ and $c+\frac{1}{a}=13$.
Find $abc+\frac{1}{abc}$.

Ans: 750

Question 25 (Factors, Counting)

The numbers from 1 to 2017 are listed in the following order: First all numbers which are *not* divisible by 3 are listed in increasing order. Then all numbers which are divisible by 3 but *not* by 3^2 are listed in increasing order. Then all numbers which are divisible by 3^2 but *not* by 3^3 are listed in increasing order, and so on. What is the last number on this list? (Give the entire number, not just its last digit.)

Ans: 1458